Instructions: Try these problems from the textbook as practice (answers below).

Problems

9-16. Evaluate the given integral by changing to polar coordinates.

9. $\iint_D x^2 y \, dA$ where D is the top half of the disk with center the origin and radius 5

13. $\iint_D e^{-x^2-y^2} \, dA$ where D is the region bounded by the semicircle $x = \sqrt{4-y^2}$ and the y-axis

15. $\iint_R \arctan(y/x) \, dA$ where $R = \{(x,y) \mid 1 \le x^2+y^2 \le 4,\ 0 \le y \le x\}$

29-37. Use polar coordinates to find the volume of the given solid.

29. Under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \le 25$.

31. Below the plane $2x + y + z = 4$ and above the disk $x^2 + y^2 \le 1$.

33. A sphere of radius a.

35. Above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

37. Inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 2y^2 + z^2 = 64$.

39-42. Evaluate the iterated integral by converting to polar coordinates.

39. $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2} \, dy \, dx$

41. $\int_0^{1/2} \int_{y\sqrt{3}}^{\sqrt{1-y^2}} xy^2 \, dx \, dy$

Answers

9. $\dfrac{1250}{3}$

13. $\dfrac{\pi}{2}(1 - e^{-4})$

15. $\dfrac{3\pi^2}{64}$

29. $\dfrac{625\pi}{2}$

31. 4π

33. $\dfrac{4}{3}\pi a^3$

35. $(\pi/3)(2 - \sqrt{2})$

37. $(8\pi/3)(64 - 24\sqrt{3})$

39. $(\pi/4)(1 - e^{-4})$

41. $\dfrac{1}{120}$